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Reg.No : .....

Name : .....

**MAHATMA GANDHI UNIVERSITY, KOTTAYAM**  
**MGU-UGP (HONOURS) Regular EXAMINATION October 2025**  
**Third SEMESTER**  
**Discipline Specific Core Course (DSC) - MG3DSCMAT201 - BUILDING**  
**BLOCKS FOR HIGHER MATHEMATICS**  
(2024 ADMISSION ONWARDS)

**Duration: 2 Hours**

**Maximum Marks: 70**

***Remember(K), Understand(U), Apply(A), Analyse(An), Evaluate(E), Create(C), Skill(S), Interest(I) and Appreciation(Ap)***

Students should attempt at least one question from each course outcome to enhance their overall outcome attainability.

**Part A**

{Short Answer Questions}

Answer any **5** questions

Each question carries **2** marks

1. What is meant by a direct proof of a conditional statement  $p \rightarrow q$ ? [K] / [CO1]
2. Let  $\mathcal{L}$  denote the set of all lines in a plane. For any two lines  $l_1$  and  $l_2$ , we write if  $l_1$  is perpendicular to  $l_2$ . Check whether "perpendicular to" is a relation on  $\mathcal{L}$ . [An] / [CO2]
3. Consider the poset  $(\mathbf{Z} \times \mathbf{Z}, \preceq)$  where  $\preceq$  is the lexicographic ordering constructed from the usual  $\leq$  relation on  $\mathbf{Z}$ . Determine whether  
(a)  $(3, 5) \prec (4, 8)$   
(b)  $(3, 8) \prec (4, 5)$   
(c)  $(4, 9) \prec (4, 11)$   
(d)  $(5, 3) \prec (4, 4)$  [U] / [CO3]
4. Construct a partition on the set  $\{a, b, c, e, f, b, i\}$ . [U] / [CO3]
5. Find the unit tangent vector of the plane curve  $\mathbf{r}(t) = t^2\mathbf{i} + e^t\mathbf{j}$ . [A] / [CO4]
6. Find principal unit normal  $\mathbf{N}$  for the space curve whose unit tangent vector  $\mathbf{T}$  is given by  $\mathbf{T} = (\frac{1}{\sqrt{2}}\sin 4t)\mathbf{i} + (-\frac{1}{\sqrt{2}}\cos 4t)\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$ . [A] / [CO4]

7. Test whether the differential form  $(x + y) dx + y dy + (y + z) dz$  is exact. [K] / [CO5]

8. Find the curl of  $\mathbf{F}(x, y, z) = xz \hat{i} + xy \hat{j} + 3xz \hat{k}$ . [A] / [CO5]

[2x5 = 10]

### Part B

{Short Essay Questions}

Answer any 5 questions

Each question carries 6 marks

9. Prove that if  $x$  is irrational, then  $\frac{1}{x}$  is irrational. [A] / [CO1]

10. (i) What do you mean by "circular reasoning"? [A] / [CO1]

(ii) Is the following argument correct? It supposedly shows that  $n$  is an even integer whenever  $n^2$  is an even integer.

Suppose that  $n^2$  is even. Then  $n^2 = 2k$  for some integer  $k$ . Let  $n = 2l$  for some integer  $l$ . This shows that  $n$  is even.

11. Determine whether the poset  $(\mathcal{P}(S), \subseteq)$  is a lattice for any set  $S$  [U] / [CO3]

12. Draw the Hasse diagram representing the partial ordering  $\{(a, b) | a \text{ divides } b\}$  on  $\{1, 2, 3, 4, 6, 8, 12\}$ . [U] / [CO3]

13. Evaluate the speed and acceleration of the position vector  $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \sin(2t) \mathbf{k}$  at  $t = \frac{\pi}{2}$  [A] / [CO4]

14. Find  $\nabla f$  if  $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z + \tan^{-1} xz$ , at  $(1, 1, 1)$ . [U] / [CO4]

15. A fluid's velocity field is  $\vec{F} = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$ . Find the flow along the curve  $\vec{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$ ,  $0 \leq t \leq 2$ . [An] / [CO5]

16. Find a potential function  $f$  for the field  $\mathbf{F} = 2xy\mathbf{i} + (x^2 - z^2)\mathbf{j} - 2yz\mathbf{k}$ . [A] / [CO5]

[6x5 = 30]

### Part C

{Essay Questions}

Answer any 3 questions

Each question carries 10 marks

17. Let  $R_1$  and  $R_2$  be relations on a set  $A$  represented by the matrices [An] / [CO2]

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \text{ Find the matrices}$$

that represent a)  $R_1 \cup R_2$ . b)  $R_1 \cap R_2$ . c)  $R_2 \circ R_1$ . d)  $R_1 \circ R_2$ . e)  $R_1 \oplus R_2$ .

18. Show that  $(\mathbb{Z}, \leq)$  is totally ordered, but not a well-ordered set. [U] / [CO3]

19. Let  $S$  a set of strings of zeros and ones. For any positive integer  $n$ , define the relation  $R_n$  on  $S$  such that  $sR_nt$  if and only if  $s = t$ , or both  $s$  and  $t$  have at least  $n$  characters and the first  $n$  characters of  $s$  and  $t$  are the same. Show that  $R_n$  is an equivalence relation. Also find three elements equivalent to 0101 in  $R_4$ . [U] / [CO3]

20. Consider the curve defined by [U] / [CO4]

$$\mathbf{r}(t) = (1 + 2t)\mathbf{i} + (1 + 3t)\mathbf{j} + (6 - 6t)\mathbf{k}$$

(a) Find the velocity vector and its magnitude.

(b) Find the arc length parameter  $s(t)$  along the curve from the point where  $t = 0$  by evaluating the integral  $s = \int_0^t |\mathbf{v}(\tau)| d\tau$ .

(c) Use the above arc length parameter to find the length of the curve for  $-1 \leq t \leq 0$ .

21. Find the line integral  $\int_C (x^2 + y^2) dy$  along the path  $C = C_1 \cup C_2$  consisting of the line segment from  $(0, 0)$  to  $(3, 0)$  followed by the segment from  $(3, 0)$  to  $(3, 3)$ . [An] / [CO5]

22. Integrate  $x + \sqrt{y}$  over the curve  $C = C_1 \cup C_2$  where  $C_1$  is the curve  $y = x^2$  and  $C_2$  is the straight line  $y = x$ . [A] / [CO5]

[10x3 = 30]